

# Quantum Histories

ADRIAN KENT

*Department of Applied Mathematics and Theoretical Physics,  
University of Cambridge,  
Silver Street, Cambridge CB3 9EW, U.K.*

## Abstract

There are good motivations for considering some type of quantum histories formalism. Several possible formalisms are known, defined by different definitions of event and by different selection criteria for sets of histories. These formalisms have a natural interpretation, according to which nature somehow chooses one set of histories from among those allowed, and then randomly chooses to realise one history from that set; other interpretations are possible, but their scientific implications are essentially the same.

The selection criteria proposed to date are reasonably natural, and certainly raise new questions. For example, the validity of ordering inferences which we normally take for granted — such as that a particle in one region is necessarily in a larger region containing it — depends on whether or not our history respects the criterion of ordered consistency, or merely consistency.

However, the known selection criteria, including consistency and medium decoherence, are very weak. It is not possible to derive the predictions of classical mechanics or Copenhagen quantum mechanics from the theories they define, even given observational data in an extended time interval. Attempts to refine the consistent histories approach so as to solve this problem by finding a definition of quasiclassicality have so far not succeeded.

On the other hand, it is shown that dynamical collapse models, of the type originally proposed by Ghirardi-Rimini-Weber, can be re-interpreted as set selection criteria within a quantum histories framework, in which context they appear as candidate solutions to the set selection problem. This suggests a new route to relativistic generalisation of these models, since covariant definitions of a quantum event are known.

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## 1. Introduction

The orthodox view of quantum theory has come under attack from several quarters lately, rather to the mystification of those who think the existing theory perfectly adequate. In particular, many can see no scientific motivation for a “consistent histories” interpretation of quantum theory[1,2,3,4,5] which apparently cannot be experimentally refuted without also refuting Copenhagen quantum theory. And, while the dynamical collapse models proposed by Ghirardi-Rimini-Weber[6] and others[e.g. 7,8,9] at least offer testable alternatives to quantum theory, their broader scientific motivations are also not widely understood.

I will try here to motivate a histories approach to quantum theory, to describe the problems (which are serious) with the existing consistent histories proposals, and to give a unifying picture in which dynamical collapse models can be re-interpreted within a history framework and seen as attempts to address the problems afflicting the existing proposals. This is not meant as a survey, but rather a personal view of some of the key ideas in the field; very different views of the consistent histories approach can be found among the papers cited. I am particularly indebted to a collaboration with Dowker,[10,11] from which some of the key points made below derive.

The existing consistent histories formulations of quantum theory have so far proved to be of little or no direct scientific use *per se*. But it seems to me that some of the motivations advanced by consistent historians are nonetheless valid and some of the questions they raise are scientifically interesting. The approach, I will argue, should be seen as an incomplete but interesting research program — a program that has run into serious problems, and whose basic assumptions now need to be reconsidered, but one that includes some natural ideas which are worth pursuing. Its ultimate aim, I will argue, must be to solve the so-called set selection problem: that is, to define a histories formulation of quantum theory from which successful existing theories — in particular, classical mechanics and the Copenhagen interpretation of quantum theory — can be derived, within their domains of validity.

It is hard to see how to produce a quantum histories formulation that is both simple and in precise agreement with standard quantum theory. But the derivation of existing theories requires only agreement with known experiment, not necessarily precise agreement with standard quantum theory. It will be shown that dynamical collapse models belong to the class of generalised quantum histories theories. Viewed in this way, they illustrate how the set selection problem can be satisfactorily solved in the non-relativistic limit.

This suggests another reason for taking collapse models seriously: in the non-relativistic case, they are the best solutions known to the problem of defining a quantum histories theory that satisfies fairly minimal scientific criteria. It seems to me also to suggest that it is more natural, and more likely to be fruitful, to look for relativistic generalisations of collapse models in the framework of history models than to try to find some form of relativistic generalisation of the stochastic differential equations that define the existing models.

## **2. The case for quantum histories**

It can be hard to look afresh at so worked-over a topic as the scientific status of quantum theory. Maybe it is helpful to recall an earlier scientific debate — that over the behaviourist program in experimental psychology.

Like many of the founders and developers of quantum theory, radical behaviourists — most notably, Skinner — took a rigorously instrumentalist view. They saw the proper task of psychology as the generation of theories predicting responses to stimuli, without invoking intermediate explanatory hypotheses about the mental states of the subject tested. Such hypotheses were, in their view, scientifically meaningless, referring as they do to unmeasurable quantities. The common language of mental states was to be understood, at best, as a sort of improper shorthand for statements about earlier stimuli. “The subject is angry”, for example, might perhaps more accurately be translated as “the subject has been repeatedly prodded with a stick”. The mind, in other words, was a black box — to be prepared for experiment by stimuli, to be investigated through its responses to further stimuli, and to be described theoretically by the correlations between the responses and stimuli.

Skinnerian behaviourism was never an unquestioned orthodoxy, and has now been almost completely abandoned. It was not rejected because of internal inconsistency, or because psychologists rejected instrumentalism on purely philosophical grounds. It simply became increasingly apparent that its axioms were obstructing good science. Mind-states are, if nothing else, useful theoretical constructs, not always, in practice, fully explicable in terms of prior stimuli. (Think of depression, for example.) And learning turns out to be a subtler process than Skinnerian accounts allow. As Chomsky showed, the acquisition of language cannot be explained purely as a product of classical stimulus-response conditioning: no scientific explanation of the fact that we speak grammatically can proceed from Skinnerian axioms alone.

The Copenhagen view of a quantum system very much resembles the Skinnerian view of the mind: statements about the system's behaviour between preparation and measurement are illegitimate. Consistent historians, like many other critics of quantum orthodoxy, reject this, taking seriously the idea that quantum events take place, whether or not anyone is looking. If we assume that the events somehow change the dynamics, clearly we have something to test. But let us suppose for the moment, as consistent historians do, that unobserved quantum events take place, but that the dynamics are unaltered. Could this sort of interpretation still lead to interesting new science?

Probably the best case that it could comes from cosmology. When cosmological ideas are discussed, we generally proceed from a quantum description of the initial or very early conditions, and then explain its consequences in terms of successive events and processes which, it is to be hoped, together explain the present state of the cosmos. Almost everyone thinks this way; almost everyone realises it is illegitimate. (Successive events? During the evolution of the closed quantum system that defines the universe? In the pre-classical era?) What can we really mean?

One possible tactic is to try to interpret all hypotheses about past events in terms of present observations. But will this always be possible? We cannot hope to calculate the probabilities of present states directly from the initial conditions in a realistic theory: any successful cosmological theory is bound to involve a long chain of reasoning involving many successive events. Practically speaking, will the present consequences of every intermediate hypothesis be calculable? And in principle, is it clear that good theories will involve only quantum events which *have* directly and independently observable present consequences? Any more, say, than it is clear that mind-states can be reduced to stimuli, in practice or in principle?

Different people have different intuitions on these questions at present. Time will tell: for what it is worth, I follow Hartle[12] in believing that the reduction will probably be impossible. Clearly, if future cosmological theories turn out to be irreducible to present observations, and if no more radical modification of quantum theory takes place, some sort of quantum histories formalism will be needed. We will need to be able to make sense of the notion that some sequence of events, drawn from some larger set of possibilities, took place during the evolution of a closed system. How then might the possible events and histories be defined?

### 3. Events and histories

A quantum event ought, presumably, to have a mathematical representation that fits naturally into a standard formulation of quantum theory, in a way that allows us to consider histories: collections of events occurring during the evolution of a system. We need, too, a natural rule for defining sample spaces of possible histories, with a probability measure. And it must be possible, at least in principle, to represent at least some familiar physical events — the results of measurements, for example — in terms of the defined histories, in order to connect the definitions with physics as we know it.

Several different definitions satisfying these demands have been proposed. The simplest version of quantum event is defined by fixing a time,  $t$ , at which it takes place, together with a Heisenberg picture projection  $P$ , which — so to speak — says what happened: the system was in the range of  $P$  at time  $t$ .

A complete list of exclusive alternative events at time  $t_j$  is then given by a projective decomposition of the identity:

$$\sigma_j = \{P_j^{(1)}, \dots, P_j^{(n_j)}\}; \quad \sum_{i=1}^{n_j} P_j^{(i)} = I; \quad P_j^{(i)} P_j^{(i')} = \delta^{ii'} P_j^{(i')}. \quad (1)$$

An elementary history  $H$  is a list of projections  $\{P_1^{(i_1)}, \dots, P_n^{(i_n)}\}$  at distinct times  $t_1$  to  $t_n$ , and a complete set of exclusive alternative histories is defined by all the possible combinations of projections from any fixed set of projective decompositions at distinct times

$$S = \{\sigma_1, t_1; \dots; \sigma_n, t_n\}. \quad (2)$$

The set  $S$  defines a sample space, and a probability distribution is defined by defining the probability of an elementary history:

$$P(H) = \text{Tr}(P_n^{(i_n)} \dots P_1^{(i_1)} \rho P_1^{(i_1)} \dots P_n^{(i_n)}), \quad (3)$$

where  $\rho$  is the initial density matrix of the system at  $t = 0$ . Note that these quantities satisfy the probability axioms — i.e. they are non-negative and add to one — without any further restriction on the set  $S$ .

This definition can usefully be generalised to “unsharp” events represented by positive operators, an idea first investigated by Rudolph.[13,14] For the discussion here the following simple definitions (not equivalent to Rudolph’s) will be adequate. An unsharp event, again at fixed time  $t$ , is defined by any positive operator  $A$ . A complete list of exclusive alternative

unsharp events at time  $t_j$  is given by a decomposition of the identity into distinct positive operators:

$$\sigma_j = \{A_j^{(1)}, \dots, A_j^{(n_j)}\}; \quad \sum_{i=1}^{n_j} A_j^{(i)} = I; \quad A_j^{(i)} \neq A_j^{(i')} \text{ for } i \neq i'. \quad (4)$$

Elementary histories and complete sets of exclusive alternative histories are defined by generalising the projection operator definitions in the obvious way, and the probability of an elementary history  $H = \{A_1^{i_1}, \dots, A_n^{i_n}\}$  is given by

$$P(H) = \text{Tr}((A_n^{(i_n)})^{\frac{1}{2}} \dots (A_1^{(i_1)})^{\frac{1}{2}} \rho (A_1^{(i_1)})^{\frac{1}{2}} \dots (A_n^{(i_n)})^{\frac{1}{2}}). \quad (5)$$

Again, these quantities automatically behave as probabilities.

Alternatively, events can be defined by partitions of the configuration space path integral.[15,12] Temporarily moving to the Schrödinger picture and taking the initial state  $|\psi\rangle$  to be pure, we can use a partition of the space of paths  $\{c_\alpha\}_{\alpha \in A}$  to define branches  $|\psi_\alpha\rangle$  and class operators  $C_\alpha$  by

$$|\psi_\alpha\rangle \equiv C_\alpha |\psi\rangle = \int_{c_\alpha} \delta q \exp(iS[q(\tau)]/\hbar) |\psi\rangle. \quad (6)$$

Summing over all paths gives the usual evolution, so that

$$\sum_\alpha C_\alpha = e^{-iHT/\hbar}. \quad (7)$$

The probability weights

$$P(c_\alpha) = \|\psi_\alpha\|^2 \quad (8)$$

can — provided  $\sum_\alpha p(c_\alpha)$  is finite — be normalised to probabilities for the elementary events  $c_\alpha$ .

From the fundamental point of view, this is perhaps the most interesting approach. Not only does it allow events to be defined covariantly in a background spacetime — the classes  $c_\alpha$  could correspond to paths crossing or not crossing various space-time regions, for example — but it can also be extended, at least formally, to sum-over-manifold formulations of quantum gravity.

It is worth mentioning for completeness (though they will not be needed here) that natural generalisations of projection-valued events corresponding to multi-time propositions can also be defined — a simple example being that the composite event defined

by the conjunction of two elementary events can be represented by the tensor product of the relevant projection operators. One of the aims of the Isham-Linden-Schreckenberg version[16,17,18] of consistent histories is to investigate general definitions of multi-time events and their physical relevance.

Logically, all of these definitions make perfect sense. Scientifically, they have two serious problems. First, whichever definition is used, probability distributions are defined on uncountably many different sets of histories, and it is not clear which of these sets (if indeed any) are appropriate in any given physical situation. Second, though the distributions obviously satisfy the mathematical axioms for probabilities, it is not at all clear that they are physically meaningful.

#### 4. “Consistency” and other selection criteria

The consistent histories approach attempts to address the last-mentioned problem, by restricting to sets of histories on which the probability distributions have properties which are, at least arguably, desirable. It is not absolutely clear, though, that these properties are always necessary for physical relevance. And it is clear that they are not sufficient: the approach does not address the first problem, as we will see.

So it should perhaps be stressed at the outset that none of the selection criteria discussed here has any privileged status. There is no logical or mathematical requirement to impose any criterion — the technical term “consistency” is here rather misleading. The various criteria proposed to date are simply guesses. Even if the basic idea of a quantum histories approach is correct, so that for any given physical system there is some identifiable set of histories which probabilistically predicts the past and future events from initial data, it is possible that this set satisfies none of these criteria.

That said, one has to start somewhere, and the various criteria do characterise interesting properties. So far as is known, not (quite) every criterion considered can be naturally extended to cover every notion of a history, but the projection operator notion of events illustrates the full spectrum of possibilities. It is simplest again to consider non-relativistic quantum theory, in the Heisenberg picture.

Griffiths’ original consistency criterion for a set  $S$  of histories is that the probability formula ought to respect the rule that the union of events can be represented by summing the corresponding projection operators, so that

$$\mathrm{Tr}(Q_n \dots Q_1 \rho Q_1 \dots Q_n) = \sum_{i_1 \in I_1 \dots i_n \in I_n} \mathrm{Tr}(P_n^{(i_n)} \dots P_1^{(i_1)} \rho P_1^{(i_1)} \dots P_n^{(i_n)}), \quad (9)$$

for all projections

$$Q_j = \sum_{i_j \in I_j} P_j^{(i_j)} \quad (10)$$

given by sums of the elementary projections at time  $t_j$  in  $S$ . This holds[1] if and only if

$$\text{Re}(\text{Tr}(P_n^{(i_n)} \dots P_r^{(i_r)} \dots P_1^{(i_1)} \rho P_1^{(i_1)} \dots P_r^{(i_r')} \dots P_n^{(i_n)})) = \delta_{i_r i_r'} p(i_1 \dots i_n), \quad (11)$$

for all  $r$  and all choices of  $i_1, \dots, i_n$  and  $i_r'$ , where  $p(i_1, \dots, i_n)$  is shorthand for the history probability (3) .

A consistent set of histories defined by positive operator events can also naturally be defined by extending equation (9) , so that we say a set  $S = \{\sigma_1, t_1; \dots; \sigma_n, t_n\}$  defined by positive operator decompositions of the form (4) is consistent if

$$\text{Tr}(B_n \dots B_1 \rho B_1 \dots B_n) = \sum_{i_1 \in I_1 \dots i_n \in I_n} \text{Tr}(B_n^{(i_n)} \dots B_1^{(i_1)} \rho B_1^{(i_1)} \dots B_n^{(i_n)}), \quad (12)$$

where

$$B_j^{(i)} = (A_j^{(i)})^{\frac{1}{2}}; \quad B_j = \left( \sum_{i \in I_j} A_j^{(i)} \right)^{\frac{1}{2}}. \quad (13)$$

The main point of these definitions is that the probability for an individual event in a history belonging to a consistent set can be calculated very simply even when one is partially or completely ignorant of past events. For example, equation (9) implies that, if nothing is known about the past, the probability of  $P_n^{(i_n)}$  would simply be

$$\text{Tr}(P_n^{(i_n)} \rho P_n^{(i_n)}); \quad (14)$$

equation (12) implies the analogous statement in the case of positive operator events.

The stronger condition of *medium decoherence*[5] requires that

$$\text{Tr}(P_n^{(i_n)} \dots P_1^{(i_1)} \rho P_1^{(j_1)} \dots P_n^{(j_n)}) = \delta_{i_1 j_1} \dots \delta_{i_n j_n} p(i_1 \dots i_n). \quad (15)$$

Again, the terminology can mislead the unwary: medium decoherence is mathematically a natural condition, but it does not generally identify sets of histories describing events characterised by decoherence in the ordinary physical sense. Gell-Mann and Hartle have also investigated[19] a criterion of *strong decoherence*. This proposal, however, must be considered exploratory: as it stands, every medium decoherent set is strongly decoherent.



These early attempts at selection criteria are very weak, in a sense to be made more precise in the next sections. One reflection of that weakness is the disconcerting fact that it is easy to find examples in which they allow two or more contrary propositions — statements corresponding to orthogonal projections — to be retrodicted from the same data, each with probability one, in different sets.[20,21] For example, one can arrange so that different sets imply that, at a given time, a given particle was in one of two different (non-intersecting) boxes. A consequence of this is that logical implications which we normally take for granted are violated. For example, according to a consistent or decoherent histories analysis, it can lead to a logical contradiction to infer from the observation that a particle was in a given region at a given time that the particle was in a larger region containing the first.

A somewhat stronger criterion, which eliminates this problem, can be defined as follows.[21] The standard partial ordering on projections, according to which  $P \leq Q$  if and only if  $PQ = QP = P$ , defines a natural partial ordering on the class of histories:

$$\{P_1, t_1; P_2, t_2; \dots; P_n, t_n\} \leq \{Q_1, t_1; Q_2, t_2; \dots; Q_n, t_n\} \Leftrightarrow P_i \leq Q_i \text{ for all } i. \quad (16)$$

Histories differing only by the inclusion of copies of the identity operator at various times are here regarded as equivalent. We now define an *ordered consistent history* to be a history  $H$ , belonging to some medium decoherent set  $S$ , with the property that if  $H'$  is a history belonging to any other medium decoherent set  $S'$  such that  $H \leq H'$  then we have  $P(H) \leq P(H')$ , and similarly  $H \geq H'$  implies  $P(H) \geq P(H')$ . An *ordered consistent set* is then a set all of whose elementary histories are ordered consistent. Ordered consistency can be defined similarly for the positive operator and path integral partition definitions of a quantum event.

There is some room for doubt as to whether ordered consistency is *too* strong a criterion: it has not been convincingly demonstrated that all familiar physics can necessarily be described by ordered consistent sets of histories. It would be particularly good to resolve this question, since either answer leads to an interesting conclusion. If arguments can be found that ordered consistent sets are adequate, then ordered consistency defines the strongest and least problematic quantum histories approach currently available that respects standard quantum dynamics. Conversely, if ordered consistent sets can be shown to be inadequate, then standard ordering implications would have to be abandoned, with radical implications for our scientific worldview: it would no longer be possible to infer

that a measurement of any observable in any range implies that it lay in any strictly larger range, for example.[21]

Two other criteria — linear positivity[22] and feasibility[23] — have also recently been defined. Both are weaker than consistency: for them to be of independent use in solving the problems considered here, some plausibly physically relevant refinement incompatible with consistency would have to be found.

To summarise, we have a spectrum of reasonably natural criteria which, as it happens, can be ordered in terms of increasing refinement: feasibility, linear positivity, consistency, medium decoherence, ordered consistency. There are thus at least six candidate quantum histories schemes, based on unrestricted sets of histories or on sets selected by one of the five criteria, and most of these schemes can be defined for each of the three natural notions of quantum event discussed to date.

Fortunately, these schemes all share some key features, which means that in assessing their present scientific status they need not all be discussed separately. Unfortunately, as we will see, this is largely because all the known criteria are far too weak.

## 5. Interpreting history-based schemes

Broadly speaking, there are two views of what the consistent histories formalism, or any other new version of quantum theory, could be good for. According to one, the idea is to understand what quantum theory really means, in some abstract idealistic sense. According to the other, the ultimate aim is to make scientific progress in the more concrete sense of generating new testable theories, allowing new calculations, and making new predictions, while retaining the successes of the Copenhagen interpretation — in short to go beyond Copenhagen quantum theory in something like the way that general relativity goes beyond Newtonian gravity. Part of the reason why the subject is so controversial, I suspect, is that it is sometimes the battleground for a kind of undeclared guerilla conflict between these motives, which perhaps are not always cleanly disentangled even in authors' own minds.

I would place recent attempts by Griffiths,[2] Omnès[24,4] — note, incidentally, that Omnès' theory of “truth”[24] is almost entirely wrong,[10] as Omnès now accepts — and Isham[25] to set out logical structures for the consistent histories formalism in the first camp. It seems to me these ideas can only be appraised on their own terms: at the moment they promise no new concrete scientific yield.

In practical terms, however, all the interpretational ideas which have been set out for the consistent histories approach have the same scientific implications, with one minor caveat that I will address in a moment. The following discussion applies equally to all the other quantum histories approaches.

Everyone agrees that the generally incompatible pictures of physics given by the uncountably many different consistent sets have to be assigned equal fundamental status. The formalism does not distinguish amongst them: to do that would need further selection criteria, which would define a different quantum histories approach. However, the physics we actually see is described by just one history.

There are three slightly different ways of interpreting the situation. The most economical is (i) that nature has chosen, somehow, one consistent set — since it is not known if there is any natural measure on the full class of consistent sets, we cannot be more precise — which defines the sample space of histories and their probabilities. Nature then randomly chooses, according to these probabilities, to realise one history, which must turn out to be the one we see.

One could say, alternatively, (ii) that one history is randomly chosen from every consistent set, or (iii) that all the histories from every consistent set are realised in numbers proportional to their probabilities. In either case, we must somehow find ourselves attached to precisely one of the realised histories. A possible attraction of these last two ways of putting things, one might think, is that they allow the possibility that the type of history we find ourselves in is not determined randomly, nor by new fundamental selection criteria, but by something to do with us — specifically, that our consciousness somehow attaches itself to quasiclassical histories. Some such hypothesis, within the second picture, seems indeed to underlie some of Gell-Mann and Hartle's and Griffiths' arguments,[5,26,10] though for obvious reasons it has not been fleshed out. However, even if these ideas could be made concrete, there is a compelling argument to show that they would not work, essentially because any given quasiclassical history belongs to many inequivalent consistent sets. This makes it impossible, in the second picture, to derive the predictions of classical mechanics or Copenhagen quantum mechanics, even under the assumption that we will persistently experience quasiclassicality.[27] (The third picture, I believe, suffers from a similar problem, though no discussion has appeared in print.)

Given this failing, and since the pictures are equivalent unless some unknown theory of consciousness is attached, we need only consider the first picture. Nature, it says, is described by one of a large number of sub-theories, which correspond to the various

sets of histories — in rather the same sort of way, for example, as general relativity says that nature is described by one of the solutions of Einstein's equations. The sub-theories in the consistent histories formalism (and the other quantum histories formalisms) are probabilistic rather than deterministic, of course, since choosing the set only determines the space of possible histories. But that itself is no drawback (except to diehard determinists). The key question is what we can achieve with this collection of sub-theories. How far can the analogy be pressed?

## 6. Why the known criteria are too weak

General relativity is almost universally seen as the paradigm of a successful physical theory, incorporating and unifying as it does special relativity, Newtonian gravity, and classical mechanics. Of course, its incompatibility with quantum theory and its singularities suggest that it will eventually be supplanted. But setting aside these problems, the theory has what might be, but usually is not, seen as an intrinsic weakness: it does not tell us *which* solution of Einstein's equations nature has chosen. This is not seen as a significant weakness since Einstein's equations can be solved locally given initial data on a hypersurface, which in turn can be approximated by carrying out measurements in a local region. We thus can and do carry out observations to determine which local solution is relevant, and hence make predictions and retrodictions within general relativity. In particular, in this way, we can derive the predictions of Newtonian gravity and classical mechanics within their domain of validity, which we understand to be the weak field limit of general relativity. In short, we understand when and why Newtonian gravity and classical mechanics hold true, and how to tell whether they will hold true in any given physical situation.

An analogously successful quantum histories approach would incorporate classical mechanics and Copenhagen quantum mechanics in a similar way. It need not provide a theory of the quantum boundary conditions — we can assume for the sake of the argument that these are fixed. Nor need it supply *a priori* the set from which nature chooses the realised history. But, applied to non-relativistic quantum mechanics, it should explain how to identify that set *post hoc*, on the basis of observations within some finite time interval. (In the relativistic case, it should presumably explain how to extrapolate a local description of the set, given observations in some finite space-time region.) And it must characterise the domain of validity of classical mechanics and Copenhagen quantum theory and explain

what types of observations are necessary in order to infer predictions and retrodictions within those theories.

No quantum histories approach defined by any of the existing criteria satisfies any of these demands — quite the reverse. For example, in any physically reasonable model, it is impossible to identify the correct medium decoherent set, or infer any of the decompositions defining its past or future events, on the basis of any set of observations taking place in any finite time interval.[10,11] If we know the initial density matrix  $\rho$  and the hamiltonian, and we observe that the series of events defined by projections  $P_1, \dots, P_n$  took place at times  $t_1 < \dots < t_n$ , we still generally cannot identify any of the projective decompositions which define the set, from which this partial history is drawn, at times before  $t_1$  or after  $t_n$ : there are almost always many incompatible medium decoherent sets which incorporate the observed data and make incompatible retrodictions of the past and predictions of the future.

In short, almost nothing can be unambiguously predicted or retrodicted on the basis of the medium decoherent histories formalism alone. We *can* make statements of the form “*if* the relevant medium decoherent set is  $S$ , then the following future (or past) events are possible (or may have occurred), with the following probabilities”. But we cannot identify  $S$ , and without doing so we cannot derive classical mechanics or Copenhagen quantum mechanics, or fully explain their successes. This is the main reason why, it seems to me, the existing quantum histories formalisms can only be viewed as part of a seriously incomplete research program. What seems to be required, if it is to be completed, is a criterion sufficiently strong that data in a finite time interval can either positively identify the relevant set of quantum histories or at least constrain the range of possibilities sufficiently that standard physics can be derived. This is the so-called *set selection problem*. [10] A set selection criterion need not, of course, necessarily be deterministic: a suitably chosen probability measure on the space of sets might do the job.

To solve the set selection problem would (almost certainly) need some mathematical characterisation of quasiclassicality — the combination of sporadic quantum unpredictability and generally deterministic classical evolution, following simple of equations of motion, that characterises our physical world. Attempts have been made to find such a characterisation by refining the consistent histories formalism.[28,19,29] The problems encountered seem formidable, and it is hard to believe a general solution will be found in the foreseeable future. Perhaps there is none.

It is sometimes necessary to step back in order to make progress. It seems at least worth considering the possibility that the criterion of consistency is too restrictive, and that the set selection problem should be addressed within the broader quantum histories framework. In fact, as the next section explains, in the non-relativistic case at least, much more progress can be made this way: dynamical collapse models can be naturally reinterpreted as candidate solutions to the set selection problem.

## 7. Unification of quantum history and dynamical reduction approaches

Ghirardi-Rimini-Weber’s “spontaneous localisation” or “quantum jump” model,[6] lucidly explained in simple terms by Bell,[30] is the ur-model of modern dynamical collapse theories. In an appropriate limit, it leads to one of a class of Markovian stochastic differential equations,[7] which define testable alternatives to the Schrödinger equation. Several concrete proposals of this type[e.g. 7,8,9,31] have been put forward, as well as more speculative ideas concerning possible relativistic generalisations.[e.g. 32,33] While this has undoubtedly been a very fruitful direction to pursue, it is surely not the only interesting way of extending the original GRW model. I would like to suggest another path here.

Recall that, according to the original GRW model, defined for  $N$  distinguishable spinless particles, the wave function

$$\psi(x_1, \dots, x_N; t) \tag{17}$$

undergoes two types of evolution. Almost all of the time, it follows the Schrödinger equation, but at discrete randomly chosen times it jumps discontinuously, so that

$$\psi \rightarrow C \exp(-(x_i - x)^2/2a^2)\psi, \tag{18}$$

where particle  $i$  is chosen randomly from 1 to  $N$ , the coordinate  $x$  is chosen randomly from the distribution

$$\int d^3x_1 \dots d^3x_N \exp(-(x_i - x)^2/a^2) |\psi|^2, \tag{19}$$

$a$  is a constant parametrising the model, and  $C$  is chosen so that the new wave function is normalised. The times of these jumps are defined by a Poisson process, with mean interval  $\tau/N$  between jumps. The parameters  $\tau$  and  $a$  are to be thought of here as new constants of nature; GRW originally suggested

$$a \approx 10^{-5} \text{ cm}, \quad \tau \approx 10^{15} \text{ sec}. \tag{20}$$

Of course, this is rather ad hoc, and no one seriously believes that these equations — or any of the models proposed to date — are likely to be precisely correct. But the GRW model and its successors demonstrate that mathematically precise theories can be found from which both the Schrödinger equation and the projection postulate can be derived as approximations,[6,30] in a way which extends to indistinguishable particles,[7] and with parameters that can be chosen consistent with experiment, so far as is known.[34]

To rephrase the model in the language of histories, note that the jump equation (18) corresponds to an unsharp event defined by a positive operator  $A_x^i$  whose action on wave functions  $\phi$  is

$$A_x^i : \phi \rightarrow \exp(-(x_i - x)^2/a^2)\phi, \quad (21)$$

so that up to normalisation (18) can be written as

$$\psi \rightarrow (A_x^i)^{\frac{1}{2}}\psi. \quad (22)$$

The operators  $A_x^i$  define a continuous decomposition of the identity:

$$\frac{1}{N\sqrt{a\pi}} \sum_i \int d^3x A_x^i = I. \quad (23)$$

The probability distribution (19) can equivalently be written as

$$\langle \psi | A_x^i | \psi \rangle = \text{Tr}((A_x^i)^{\frac{1}{2}} |\psi\rangle \langle \psi| (A_x^i)^{\frac{1}{2}}). \quad (24)$$

(This is no accident: GRW's definitions were motivated by the theory of unsharp measurements.)

Now, if the initial state  $\rho = |\psi\rangle \langle \psi|$ , then the probability (5) for the quantum history defined by a series of unsharp events chosen from decompositions (23) is

$$\text{Tr}((A_{x_n}^{i_n})^{\frac{1}{2}} \dots (A_{x_1}^{i_1})^{\frac{1}{2}} \rho (A_{x_1}^{i_1})^{\frac{1}{2}} \dots (A_{x_n}^{i_n})^{\frac{1}{2}}). \quad (25)$$

Translating from the Heisenberg picture to the Schrödinger, we see that (18) and (19) are precisely the outcomes and probabilities for unsharp events of this kind.

Rewritten in this way, the GRW model defines a probabilistic set selection rule for a quantum histories formulation based on unsharp events. The set is selected by the choice of decompositions (23) together with the random choice of Poisson times; its histories are given by sequences of unsharp events  $A_x^i$  at the chosen times. Since continuous stochastic equations of quite general form can be arbitrarily well approximated[7] by discrete jump models of GRW type, the later dynamical collapse model proposals[7,8,9,31] can also effectively be interpreted in the same way. The selected sets, however, violate (12) and so are not consistent — which is why the models disagree with standard quantum theory. If one is willing to pay this price (without necessarily contradicting experiment), quasiclassicality is not so hard to characterise.

## 8. Conclusions and prospects

Things happened in the past, which were unobserved at the time, and whose consequences it is now impractical to describe in terms of present observations; to understand the present state of the world properly, we need to be able to include such past events in our theories — these may not be unquestionable assumptions, but they do not seem particularly outlandish. They could turn out to be more or less forced on us by accumulating cosmological data.

In any case, it seems worth trying to incorporate them into quantum theory. The least radical way of doing so is to try to find a natural representation of past events in some standard approach to quantum theory — perhaps as projections, positive operators, or partitions of the path integral — and then to try to define some probabilistic interpretation in which histories of events are the primary objects. This leads naturally to some form of quantum histories approach, in which histories are grouped into complete sets of exclusive alternatives, on each of which sets a probability measure is defined.

One can then, by trying to characterise interesting mathematical properties of sets of histories, try to develop criteria which select out sets that might be particularly physically interesting. Any criterion defines a new quantum histories formalism; all of these formalisms have a natural interpretation. This is the route pioneered by Griffiths,[1] Omnes,[3] Gell-Mann and Hartle,[5] who have set out a consistent (or decoherent) histories interpretation of quantum theory based on particular choices of criteria; stronger[21] and weaker[22,23] natural criteria have also been found.

As a research program, the quantum histories approach has been, and presumably will continue to be, very productive, raising many new and interesting questions. However, considered as a finished product, the consistent (or decoherent) histories interpretation must, I believe, be judged a failure as a scientific theory. Its relation to classical mechanics and Copenhagen quantum mechanics is very different from, for example, that of general relativity to Newtonian gravity and fluid dynamics. And the comparison is not to its advantage: it is unable to account for the simplest predictions or retrodictions, or to explain the success of Copenhagen quantum mechanics or classical mechanics.[10,11]

Even judged as mathematical criteria, consistency and medium decoherence, while undoubtedly interesting properties of sets of histories, involve arbitrary choices and have some decidedly unnatural features.[20] It may, in any event, be more sensible to treat the existing criteria as useful taxonomic labels rather than as badges of validity. Perhaps some



inconsistent sets of histories will turn out to be scientifically useful; certainly almost all consistent sets will not.

The key scientific problem in quantum histories approaches is to find some set selection rule, probabilistic or deterministic, sufficiently strong that it allows classical mechanics, Copenhagen quantum mechanics, and quantum field theory to be derived within characterisable domains of validity. (Given a quantum theory of gravity, one would similarly hope to be able to derive general relativity.)

It is an open question whether any precise rule of this type can be found within the consistent histories approach, even in the non-relativistic case. The attempts to date do not inspire overwhelming optimism. However, by going outside the consistent histories framework, and deviating from standard quantum mechanics, a solution to the non-relativistic set selection problem can be found, by reinterpreting dynamical collapse models of Ghirardi-Rimini-Weber type in the framework of quantum histories.

Encouragingly from the point of view of relativistic generalisation, the quantum histories framework includes covariantly defined notions of event.[12] In this sense each approach seems to hold out the prospect of a solution to the deepest problem of the other. A covariantly defined set selection rule, which picks out generally inconsistent sets and reduces to something resembling a dynamical collapse model in the non-relativistic limit, would be a particularly attractive way of solving the deep problem of interpreting quantum theory in the cosmological context, since it need not necessarily require any great conceptual revolution that threatens the successes of our present theories or (most of) their fundamental principles. It would, of course, disagree at least subtly with the predictions of standard quantum theory — but then, if nature really has chosen to make fundamental use of the notion of a quantum event, it would seem uncharacteristically tasteless to have done so in a way that leaves such events entirely undetectable.

Though the line of thought which leads to this last speculative proposal could, of course, be wrong in any of several places, it seems to me to strengthen the case for taking dynamical collapse models seriously.

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